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Research Article

Mathematical Modelling: A Retrospective Overview

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Abstract

This study aims to comprehensively view the scientific articles published on mathematical modelling (MM) before 2023. In this context, analyzed articles published on MM with bibliometric analysis under four main headings; scientific productivity, network analysis, conceptual structure, and thematic map. The Web of Science database was used to analyze 906 articles published by 2039 authors representing 68 countries from 1981 to 2023. According to the study's findings, the articles published on MM differ yearly, but the number of citations is constantly increasing. Erbas, A. K., Schukajlow, S., and Kaiser, G. are the most productive authors. The most productive institutions are Purdue, Australian Catholic, and Hamburg Universities. According to the network analysis, the journals ZDM Mathematics Education and Educational Studies in Mathematics come to the fore. It was determined that the best size reduction obtained in the conceptual analysis constituted approximately 44% of the total variability. According to the findings obtained at the end of the research, made some suggestions.

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Introduction

Mathematical modelling (MM) arising from the needs of daily life has a history as old as mathematics (Dost, 2019). However, its importance and value in mathematics teaching have been better understood after the innovative curriculum changes experienced around the world towards the end of the 1990s (Lingefjärd, 2006). The inadequacy of traditional mathematics teaching approaches in developing basic mathematical skills such as problem-solving, reasoning, associating with daily life, and critical thinking that individuals will need has accelerated the emergence of MM studies (Blum, 2011; Borromeo-Ferri, 2013; Common Core Standards Writing Team [CCSWT], 2013; English & Watters, 2004; National Council of Teachers of Mathematics [NCTM], 2000; Stillman, Kaiser & Lampen, 2020). In this direction, MM, which is believed to play a critical role in mathematical practice and to improve learning, has been integrated into many mathematics curricula (Common Core State Standards for Mathematics [CCSS-M], 2010; Ministry of National Education [MoNE], 2018).

With this understanding, the idea of the necessity of teaching mathematical concepts that reflect real-world has taken its place among the primary objectives of the mathematics curriculum of many countries and has enabled vital steps to be taken in this direction (MoNE, 2018; NCTM, 2016). So much so that the calls for taking modern subjects into account in mathematics education, which were frequently voiced in the past years, were answered by MM (Blum, 2011; Greefrath, Siller, Vorhölter & Kaiser, 2022). MM's popularity has increased in the last twenty years and has become an essential field of study in today's mathematics education. MM competencies have been at the centre of national curricula, with the contribution of research that focuses more on this area (Blum, 2015; Vorhölter, Greefrath, Borromeo-Ferri, Leiß & Schukajlow, 2019). Because modelling and modelling applications are the essential components of mathematics, as well as the findings of many studies that offer valuable opportunities to apply mathematical knowledge in real-life (Cevikbas, Kaiser & Schukajlow, 2021; Erbas, Kertil, Cetinkaya, Alacacı, Cakıroğlu & Bas, 2014). As a matter of fact, in today's education systems, MM is included in detail in the context of mathematics teaching programs, which are applied at all levels from primary school to higher education in many countries such as Germany, America, Australia, Finland, Switzerland, Sweden, Singapore, China and Turkey (Bukova-Güzel, 2021). MM has become a necessity in various disciplines, not only in mathematics but also biology, engineering, finance, computer science, and social sciences (Cirillo, Pelesko, Felton-Koestler & Rubel, 2016). Especially in order to keep up with the rapid changes in societies, the increasing need for individuals who are compatible with technology, who can think creatively, and who has a flexible and critical perspective has led nations to raise individuals equipped with MM competence and have led to sharp evolutions in this direction (Lingefjärd, 2006). One of the main reasons for these evolutions is that MM, unlike other forms of application of mathematics, predicts an apparent reconciliation between mathematics and the real-life situation and that the results are mathematically correct and reasonable in the context of real-life (Pollak, 2003). Therefore, by encouraging the use of MM in teaching mathematical concepts that reflect real-life, the door to make mathematics education meaningful has been opened (Stillman et al., 2020).

Although there are many different definitions of the concept of MM in the related literature, it is seen that all definitions come together as a mathematical expression of an actual situation (CCSWT, 2013; Haines & Crouch, 2010; Lesh & Doerr, 2003). MM is generally considered the process of analyzing a real-life or realistic situation using mathematical methods (Erbas et al., 2014). In other words, it is "using mathematics or statistics to explain a real-life situation and to extract additional information about the situation through mathematical calculation and analysis" (CCSWT, 2013, p. 5). On the other hand, MM is considered a mathematical process that includes observing a phenomenon, estimating relationships, applying mathematical analyses (equations, symbolic structures, *etc.*), obtaining mathematical results, and reinterpreting the model (Swetz & Hartzler, 1991). According to Lesh and Doerr (2003), MM is a process in which existing conceptual systems and models are used to create and develop new models in new contexts. Therefore, mathematical models are frequently used to clarify a phenomenon, interpret it, and solve problems (NCTM, 2014). In general, modelling is a creative process of making sense of real-life to define, control or optimize aspects of a situation (Kaiser, 2017, 2020). This consensus requires a comprehensive understanding of the concept of MM and teaching and learning modelling (Xu, Lu, Yang & Bao, 2022). In this context, researchers have presented and applied many theoretical approaches regarding the content of the concept of MM (Frejd & Bergsten, 2018; Niss & Blum, 2020). For example, in the approach conceptualized by Blum (2015), the modelling perspective, the purposes of modelling teaching, and examples suitable for the teaching content and modelling process cycles are included. These approaches are; applied to model, educational modelling, socio-critical modelling, pedagogical modelling, epistemological modelling, and conceptual modelling. Similarly, realistic and applied models adopting the pragmatic perspective, educational modelling related to scientific and humanistic perspectives, and socio-critical and socio-cultural modelling perspectives emphasizing the role of mathematics in society were stated as the primary approaches by Kaiser and Sriraman (2006). Although there are many theories and approaches to MM, there is no agreed definition of MM, nor is there an agreed modelling cycle. Instead, created MM cycles are the authors' attempts to capture the essence of a creative and dynamic process (Cirillo et al., 2016).

MM has taken an important place in many disciplines from the past to the present. When the studies on the subject of MM are examined, it is seen that various studies on the MM competence and skills of students, teacher candidates, and teachers come to the fore. These studies focused on the effects of practices based on MM skills on students' conceptual learning (Blomhøj & Jensen, 2003; Cetinkaya, Kertil, Erbaş, Korkmaz, Alacacı & Cakıroğlu, 2016; Çavuş-Erdem & Gürbüz, 2018; Galbraith, 2012; Greefrath et al., 2022; Haines & Crouch, 2010; Kaiser & Sriraman, 2006; Kertil, Erbas & Cetinkaya, 2017; Lesh & Lehrer, 2003; Maaβ, 2006; Michelsen, 2006; Verschaffel, Greer & De Corte, 2002; Xu et al., 2022). In addition to these, success based on MM competencies (Bukova-Güzel & Uğurel, 2010; Hıdıroğlu, Tekin-Dede, Kula-Ünver & Bukova-Güzel, 2017; Kandemir, 2011; Kaya & Keşan, 2022; Sokolowski, 2015), high-level thinking skills and problem-solving (Didis, Erbas, Cetinkaya, Cakıroğlu & Alacacı, 2016; English, Fox & Watters, 2005; Kim & Kim, 2010; Lesh & Harel, 2003; Niss & Blum, 2020; Niss & Højgaard, 2019; Schukajlow, Kolter & Blum, 2015) creativity, awareness, performance, and motivation (Akgün, Çiltaş, Deniz, Çiftçi & Işık, 2013; Bora & Ahmed, 2019; Bukova-Güzel, 2011; Daher, 2021; Lu & Kaiser, 2021; Novak, Daday & McDaniel, 2018; Sriraman, 2009; Wessels, 2014; Zbiek & Conner, 2006) and other topics were also discussed. When studies conducted in a similar direction were investigated in the literature, it was determined that there were a limited number of studies and little overlap with the study carried out. For example, in Cevikbas et al. (2021), the full texts of 75 studies indexed in well-known databases and published in English were analyzed. As a result of the research based on the conceptualization of modelling competencies, it has been determined that the studies on theoretical approaches to MM increased significantly in 2020. In addition, although the studies focused on analysis tests, it was determined that the methods used to measure modelling competencies were rich in content. In another study, when the contents of the postgraduate theses on MM made in Turkey by Yıldız and Yenilmez (2019) were examined, it was determined that the studies were primarily conducted in mathematics teaching and primary school mathematics teaching. After 2005, it was determined that the number of studies increased, and more studies were conducted with middle school students and teacher candidates. It has been reported that qualitative and mixed methods are primarily used, and opinions on modelling are given more place. In another study conducted by Birgin and Öztürk (2021), a total of 160 studies (63 master's theses, 26 doctoral theses, and 71 articles) on mathematical modelling in Turkey between the years 2010-2020 were examined. The study has determined that mathematical modelling studies have increased since 2016 in Turkey, qualitative research methods are preferred more, and case study and experimental research designs are used more. In addition, it was determined that many studies focused on the analysis of the MM process and environment, the determination of the participants' views and competencies regarding modelling skills, and the effect of MM-based teaching on student achievement and modelling skills. In the study by Aztekin and Şener (2015), in which studies on MM were examined through meta-synthesis, it was determined that MM studies conducted in Turkey did not reach a sufficient level of scope and diversity. In addition, it was determined that most of them used the case study method, and mainly pedagogical objectives were observed.

Today, MM is seen as an essential skill that needs to be developed in students for both mathematics and other disciplines (Blomhøj & Kjeldsen, 2006; Blum, 2011; English et al., 2005; Gainsburg, 2013; Kaiser & Brand, 2015; Lesh, & Doerr, 2003; Stillman et al., 2020). One of the aims of teaching mathematics is to enable individuals to acquire and use the modeling skills they need to solve problems related to real-life situations. Therefore, knowing the contents of the studies on MM is very important for new studies. Especially noticing the changes in scientific research on MM from past to present and drawing a general framework are guiding future studies. For these reasons, the effectiveness of MM in the WoS database is discussed in the context of bibliometric indicators in this study. Thus, it is aimed to determine the international view of the studies on MM. It is thought that the findings of this study will provide a different perspective and an essential resource for researchers who plan to research the subject of MM. The fact that there is no study conducted in a similar direction in the relevant literature reveals the necessity of this study and aims to fill this gap in the field. The study tried to determine a comprehensive view of the scientific articles published on the subject of MM in the WoS database. In this direction, articles published before 2023 were evaluated in terms of scientific productivity, network analysis, conceptual structure, and thematic approaches. In the study, answers to the following questions were sought:

RQ 1. What are the publication and citation patterns of the MM topic?

RQ 2. Which authors, institutions, and nations contribute the most to the MM?

RQ 3. Which journals are closely linked to the topic of MM?

RQ 4. Which authors, institutions, and nations are involved in collaborative work on MM?

RQ 5. What are the trending topics/themes in research on MM?

Method

Research Model

The primary purpose of this research is to determine the retrospective view of scientific articles published on MM before 2023. A bibliometric methodological process was followed to conduct the study and seek answers to the identified research questions.

Bibliometrics, by its nature, means combining, managing, and researching bibliographic information obtained from publications with scientific content (Verbeek, Debackere, Luwel & Zimmermann, 2002). Bibliometric analysis; is an effective technique used to summarize a field's scope and intellectual structure by analyzing the social and structural relationships between different research components (author, country, institution, subject, *etc.*) (Donthu, Kumar, Mukherjee, Pandey & Lim, 2021). This analysis technique is a dynamically evolving method for evaluating research results and comprehensively capturing scientific outputs (Grzybowska & Awasthi, 2020). The most distinctive feature of this scientometry is its focus on the potential to search, record, analyze and predict processes in the scientific literature (Martínez, Cobo, Herrera & Herrera-Viedma, 2015). This way the scientific subjects' qualitative and quantitative changes are determined. It also enables statistical methods to create a general profile of publications on a chosen subject or identify trends in a discipline (De Bakker, Groenewegen & Den Hond, 2005). This analysis can evaluate published materials (books, articles, book chapters, *etc.*) for the research topic (Jing, Qinghua & Landström, 2015). In particular, it allows the co-occurrence of keywords or meaningful terms and the formation of a network based on the relationships between items (Cheng, Wang, Mørch, Chen & Spector, 2014). In the study, bibliometric data were extracted related to the subject of MM and discussed their conceptual, intellectual, and social structures. This way, descriptive, cross-sectional-retrospective bibliometric results are presented by analyzing scientific documents published on MM. Accordingly, the methodological process followed in the study is as follows: (i) identifying research questions (problems or sub-problems), (ii) identifying relevant studies, (iii) selecting studies, (iv) creating a datasheet and collection, (v) summarize, report and discussion stages. The processes followed to increase the overall reliability of the determined framework and to indicate the transparency of the study are discussed in detail in the following sections.

Data Collection

The Web of Science™ Core Collection database was used to create a source database on MM. This database is a multidisciplinary database within Clarivate Analytics. The WoS™ database platform has 254 subject categories, 12 million peer-reviewed full-text open-access editions, more than 170 million records, more than 34,600 journals, more than 4,900 publisher partners, and more than 1.9 billion cited reference information (Web of Science Group [WoSG], 2022). Thanks to this large-scale platform, the world's most comprehensive source of academic knowledge, it allows searching multiple databases simultaneously from a single interface through the WoS™ Core Collection. The most important feature of the WoS™ database is that it is reliable, fast and easily accessible, organized, and self-consistent (Zhao & Strotmann, 2015). On the other hand, it allows researchers to extract bibliographic information about selected records, providing an ideal data source for conducting scientific studies (Fang, Zhang & Qiu, 2017). It also helps researchers improve their results. It offers valuable records for analyzing productive authors, journals, and institutions in studies in a particular field. Within the scope of the study, scientific studies on MM were selected from the database through a search term. Bibliometric network analyses were carried out immediately after the preliminary examinations. Finally, the process was completed with conceptual structure and thematic mapping processes. The procedure followed in the structuring of the research is summarized as follows.

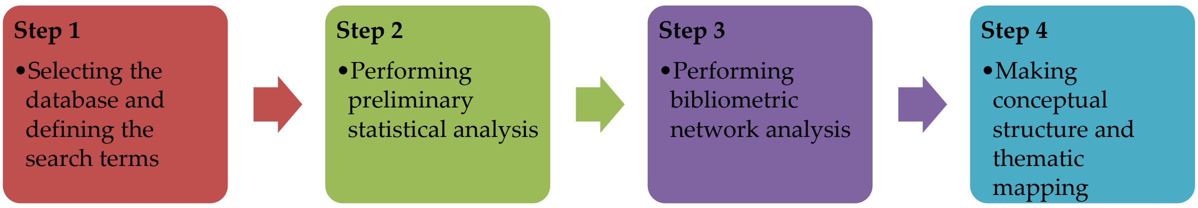


Figure 1. Process followed in the study

In this study, all periods before 2023 were determined to collect the highest number and best quality articles on the subject of MM in the database. Only articles written in English were preferred in the study. Mathematical modelling was selected as the search term, Social Sciences Citation Index [SSCI], Science Citation Index Expanded [SSCI-Expanded], Emerging Sources Citation Index [ESCI], and Arts & Humanities Citation Index [A&HCI] were preferred as WoS Index. Education & Educational Research and Education Scientific Disciplines were chosen as WoS categories. In the study, only scientific articles were used as the document type.

Data Analysis Process

The study used scientific articles in the WoS™ Core Collection database for data analysis. "Mathematical modelling" was used in the data search, and an effort was made to reach more scientific articles. The encoding written to the scanning module is: Documents Topic = (math* model*). Pre-scanning was done without restrictions in the scanning module, and 306.119 scientific contents were reached. When the WoS categories were chosen as Education & Educational Research and Education Scientific Disciplines without any time-frame restrictions, it was determined that there were 2359 records. When the document type article was selected, 1215 article records were reached, and when the English language was decided, 1058 article records were gone. Finally, when SSCI, SSCI Expanded, ESCI, and A&HCI were selected as WoS indexes, 906 scientific articles were accessed, and final data were obtained. These last data are saved as Plain Text Files. R 4.2.2 software, a version of RStudio, which includes various libraries such as bibliometrics, word cloud, and ggplot2, was used to perform the analysis (Aria & Cuccurullo, 2017). VOSviewer 1.6.18 program was used for network visualization of the obtained data. VOSviewer is software for creating and visualizing maps based on network data. It is also highly functional for viewing and interpreting large bibliometric maps (Van Eck & Waltman, 2019). Table 1 contains the preliminary information about the data obtained on the subject of MM.

Table 1. Main information about data

Description	Results	Description	Results
Main information about data		*Document Contents*	
Timespan	1981:2022	Keywords plus (ID)-Author's keywords (DE)	946-2473
Sources (journal, book, *etc*)	194	*Authors- Authors Collaboration*	
Document	906	Authors	2039
Annual Growth Rate %	11.03	Authors of single-authored docs	191
Document Average Age	8.64	Single-authored docs	215
Average citations per doc	8.29	Co-Authors per doc	2.7
References	23394	International co-authorships %	14.79

According to Table 1, 906 research articles were handled by 2039 authors. There is 23394 reference information in the studies. While the number of pieces with a single author was 191, the remaining articles had more than one author. The cooperation index among the authors is 2.7, and the percentage of international cooperation is 14.79. Within the scope of the study, the thematic map and strategic diagram analyses proposed by Law, Bauin, Courtial and Wittaker (1988) are also included. Thematic maps reveal dynamic cluster formations by analyzing keywords or co-axial word formations (Gonzales-Valiente, 2019). Conceptual structure maps are used to investigate the conceptual structure of the research topic by dividing the determined research topic content into knowledge clusters (Wetzstein, Feisel, Hartmann & Benton, 2019).

Findings

This section presents the findings based on scientific production, network analysis (co-citation networks, keywords, co-occurences network analysis, trending topics and thematic evolution), and conceptual structure and thematic maps. Findings for sub-problems are presented under headings.

Scientific Production

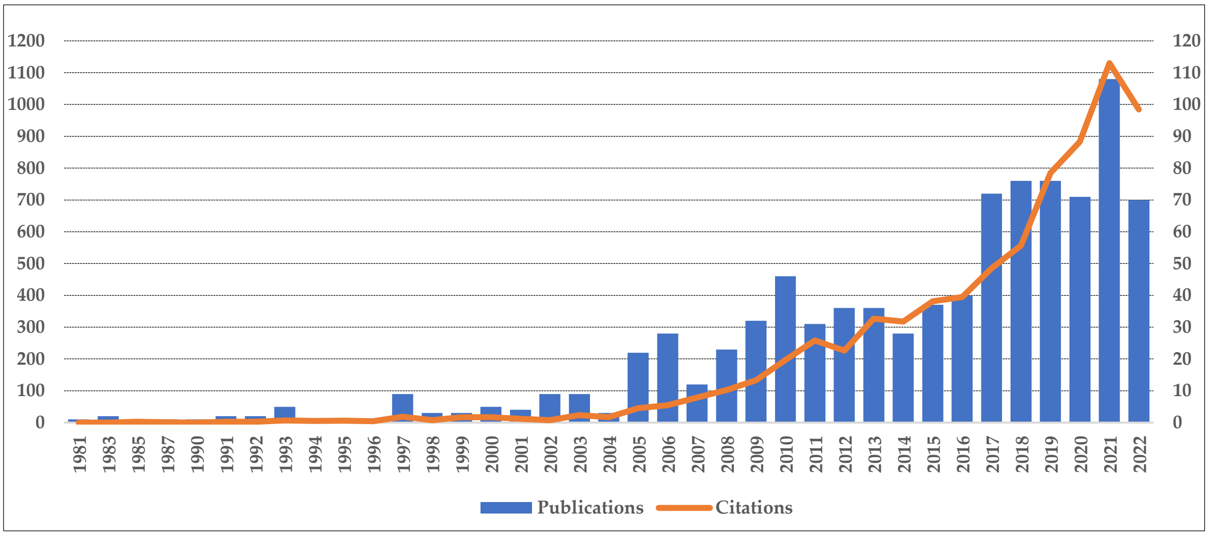


Figure 2. Annual scientific production and citation

Figure 2 shows the data of scientific outputs on MM. The number of studies published on MM has varied considerably over the years. There was no noticeable increase in the number of studies published on MM between 1981 and 2005. Although there was an increase in 2006, there was a sharp decrease in the following year. Between 2008 and 2016, it published between 23 and 40 articles, and there was little increase between these years. With 2017, it is striking that there has been a significant increase in MM compared to other years. On the other hand, the rise in the number of citations has continued to increase since 2006. The increase in the number of citations due to the rise in scientific publications on MM indicates that it is likely to continue. Below are the core authors who have published scientific articles on MM (Table 2).

Table 2. Most productive "core" authors

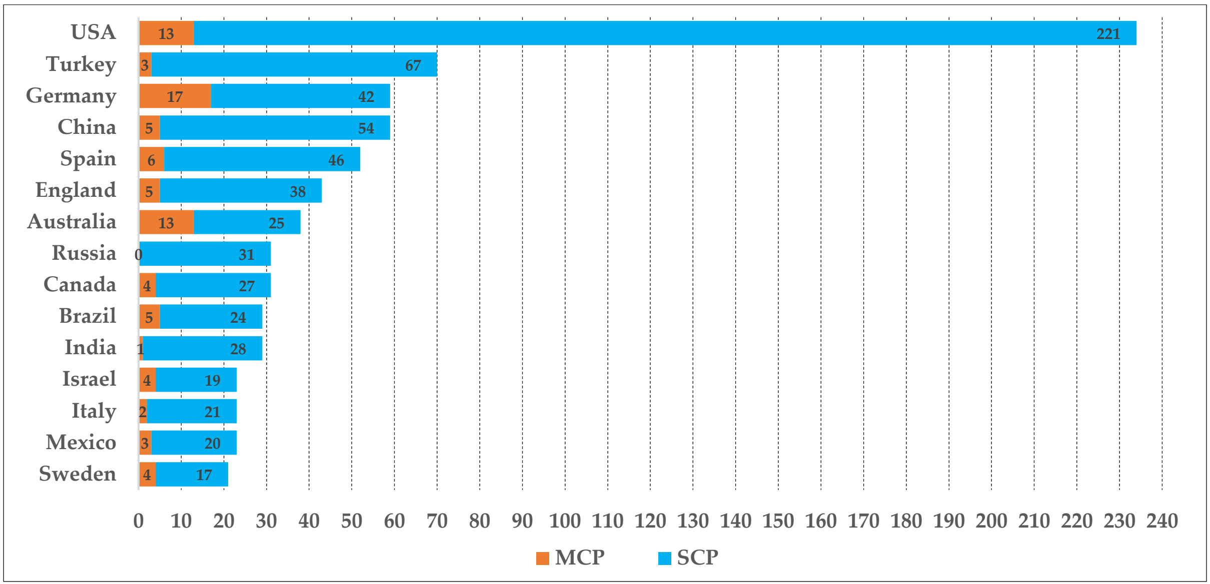
Authors	Article	Articles Fractionalized
Erbas, A. K.	13	3.80
Schukajlow, S.	12	3.53
Kaiser, G.	11	4.17
Verschaffel, L.	10	3.08
Cetinkaya, B.	9	2.13
Kertil, M.	7	2.50
Alacacı, C.	6	1.13
Albarracin, L.	6	3.25
Becker, N. M.	6	1.35
Çakıroğlu, E.	6	1.13
Czocher, J. A.	6	3.92
Leiss, D.	6	2.07
Maass, K.	6	1.95
Van Dooren, E.	6	1.58

When Table 2 is examined, the core authors who have published scientific articles on MM have published 110 articles, constituting approximately 12% of all publications. In the study, Egghe's (1987) formula determines used the minimum number of publications of a core author (0.749*√Pmax, Pmax is "the maximum number of articles published by an author in the field"). Erbas, A. K. has the highest number of articles, so the Pmax is 13, and the minimum number of articles a core author should publish is 6. The most productive authors are Erbas, Schukajlow, Kaiser, Verschaffel, and Cetinkaya. The most cited articles are listed below (Table 3).

Table 3. Most cited articles (by total citation per year) published in MM

Paper	Doi	Total Citations	TC per Year	Normalized TC
Carlson, M., 2002	10.2307/4149958	232	10.55	5.04
Greer, B., 1997	10.1016/S0959-4752(97)00006-6	162	6.00	2.91
Huang, S., 2013	10.1016/j.compedu.2012.08.015	152	13.82	10.67
Artigue, M., 2013	10.1007/s11858-013-0506-6	115	10.45	8.07
Volpe, G., 2014	10.1119/1.4870398	111	11.10	9.03
Reusser, K., 1997	10.1016/S0959-4752(97)00014-5	109	4.04	1.96
Doerr, H. M., 2003	10.2307/30034902	101	4.81	6.27
Redish, E. F., 2008	10.1002/j.21689830.2008.tb00980.x	94	5.88	6.49
Smith, R., 2010	10.1177/1046878109334330	92	6.57	7.21
Greca, I. M., 2002	10.1002/sce.10013	92	4.18	2.00
Verschaffel, L., 1997	10.1016/S0959-4752(97)00008-X	85	3.15	1.53
Verschaffel, L., 1997	10.2307/749692	75	2.78	1.35
Morio, J., 2011	10.1088/0143-0807/32/6/011	65	5.00	5.13
Precup, R. E., 2011	10.1109/TE.2010.2058575	62	4.77	4.89
Valdez, P., 2008	10.1111/j.1751-228X.2008.00023.x	60	3.75	4.14
Burkhardt, H., 2006	10.1007/BF02655888	59	3.28	3.20

Table 3 shows the authors most cited due to their published articles. The most cited articles in the table are Carlson (10.55 citations per year), Greer (6.00 citations per year), Huang (13.82 citations per year), Artigue (10.45 citations per year), Volpe (11.10 citations per year), Reusser (4.04 citations per year), Doerr (4.81 citations per year) and Redish (5.88 citations per year) by the author(s). Published articles were conducted with multiple authors and a single author. The distribution of the related authors according to their institutions is given in the figure below (Figure 3).



Notes: MCP = Multiple Country Publications; SCP = Single Country Publications

Figure 3. Articles published in the MM by corresponding author's country

When Figure 3 is examined, it is seen that the authors working on MM are mostly related to various countries such as the USA, Turkey, Germany, China, Spain, England, and Australia. It is noteworthy that the authors are generally limited to a single country while conducting their studies. For example, out of 234 USA-related articles, 221 include authors from one country and 13 from more than one country. Similarly, 67 of the 70 articles related to Turkey are from a single country, while only 3 include authors from more than one country. All 31 Russia-related articles contain a single-country author. The study consists of authors from more than one country and is found in articles related to Germany. While 42 of the 59 articles related to Germany had a single country author, 17 included multiple country authors. The figure below gives information about the authors who played a dominant role in MM (Figure 4).

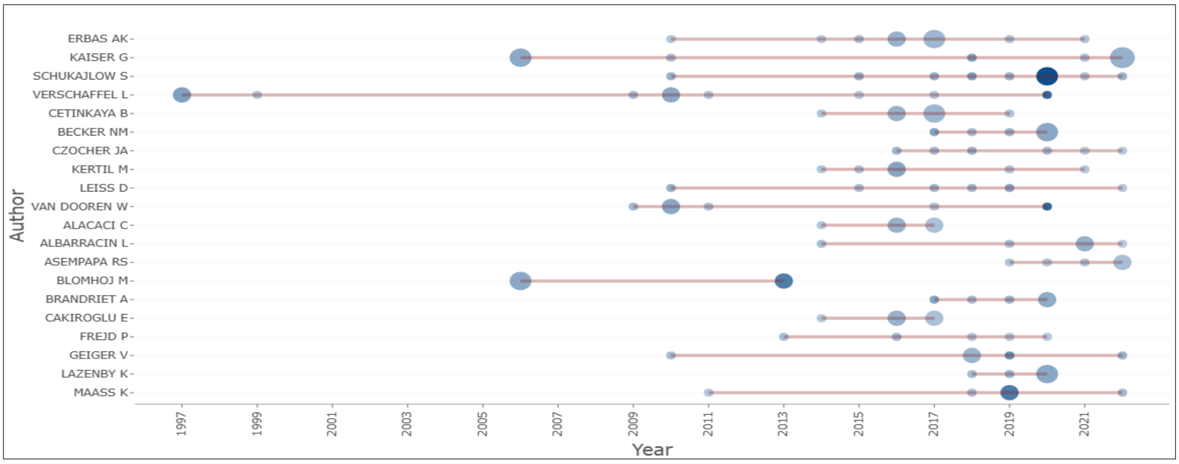


Figure 4. MM author dominance over the time

Figure 4 shows the dominant authors over time. The dominance factor is a bibliometric measure that calculates author dominance by dividing the number of multi-authored articles by the total number of multi-author articles (Kumar & Jan, 2014). When Figure 4 is examined, Erbas from 2010 to 2021, Kaiser from 2006 to 2022, Schukajlow from 2010 to 2022, and Verschaffel from 1997 to 2020 played dominant authors. On the other hand, some authors have assumed a short-term dominant role. For example, the dominance of the author named Czocher lasted from 2016 to 2022, the dominance of the author named Çakıroğlu from 2014 to 2017, and the dominance of the author named Blomhoj from 2006 to 2013. The distribution of productive institutions related to MM studies is given below (Figure 5).

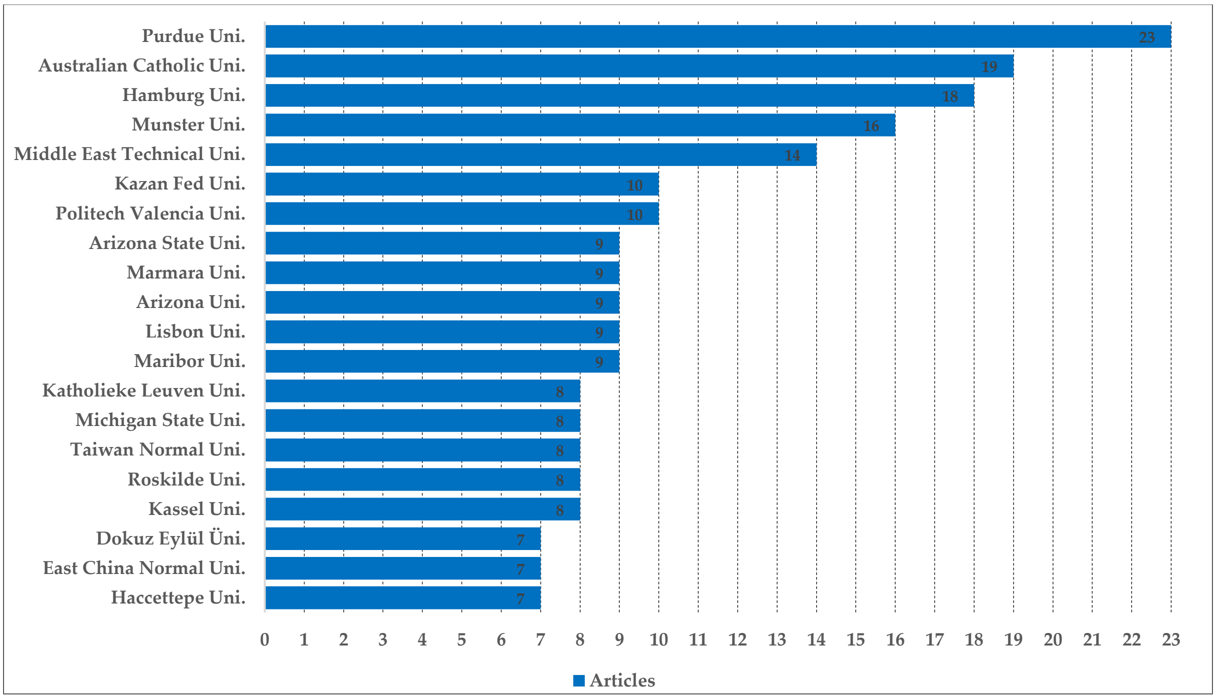


Figure 5. Most relevant affiliations producing MM research

When Figure 5 is examined, it is seen that the most productive institution in MM is Purdue University, with 23 articles. Australian Catholic follows this with 19 articles, Hamburg with 18, Munster with 16, Middle East Technical with 14, and Kazan Fed and Politech Valencia Universities with 10, respectively. The geographic atlas of the articles are presented below (Figure 6).

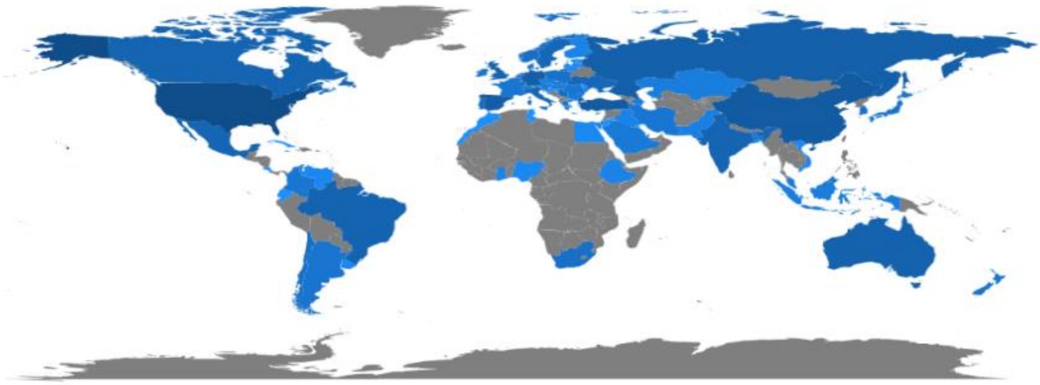


Figure 6. Country scientific production (darker shades indicate more scientific production)

When Figure 6 is examined, USA, China, and Turkey are at the top of the scientific production list with 426, 123, and 120 documents, respectively, and are shown in bold. Other countries include Spain (89), Germany (86), Russia (74), Australia (59), the United Kingdom (54), Brazil (51), Canada (49), and India (46). When the geographical atlas of MM is examined in general terms, it is seen that there are documents from many continents.

Network Analysis-Co-Citation Networks

Co-citation analysis is defined as the frequency of citing two studies together. This analysis technique shows the citation frequency of two analysis units in the same research (Bağış, 2021, p. 100). In other words, it is a visualization of the frequency of citing two studies together. The network visualization of co-citation analysis is included in the context of the cited authors in the figure below (Figure 7).

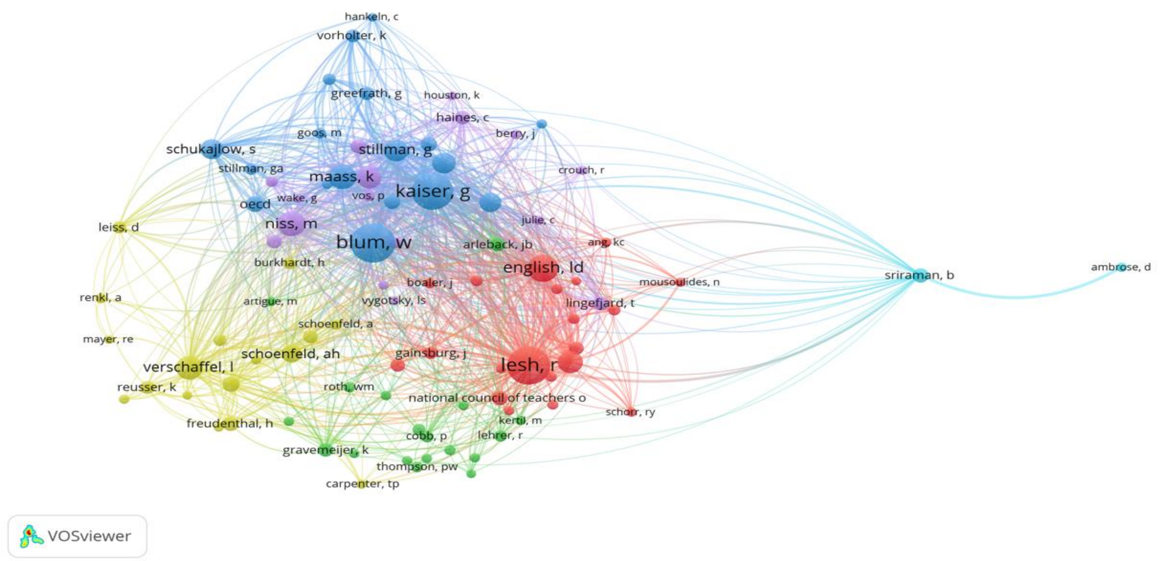


Figure 7. Co-cited network analysis in the context of cited authors

In Figure 7, the articles published on MM have been chosen so that the minimum number of citations of one cited author is twenty. According to this threshold value, the total number of cited authors from the studies in the data set made the analysis 16293, and the number of cited studies meeting the threshold value is 91. Depending on the color used, six different clusters emerged in the figure. The cluster in red includes authors such as Lesh, Doerr, English, and Gainsburg. In this cluster, Lesh [number of connections: 89, total connection strength: 5699] is central. The authors of English [number of connections: 85, total connection strength: 3075] and Doerr [number of connections: 86, total connection strength: 2130] also have strong connections in this cluster. The green cluster includes authors such as Cobb, Ärlebäck, Lehrer, and Gravemeijer. The author who dominated this cluster was Ärlebäck [number of connections: 78, total connection strength: 1117]. The authors of Gravemeijer [number of connections: 75, total connection strength: 781] and Cobb [number of connections: 76, total connection strength: 656] also have strong connections. The authors of Blum, Kaiser, Borromeo, Ferri, Stillman, and Maass come to the fore in the blue cluster. Blum [number of connections: 88, total connection strength: 6393] and Kaiser [number of connections: 81, total connection strength: 6172] stand out in this cluster. Other than that, the authors of Stillman [number of connections: 86, total connection strength: 2330] and Mass [number of connections: 88, total connection strength: 3042] also have strong connections. In the yellow cluster, it is seen that the author of Verschaffel [number of connections: 84, total connection strength: 2238] dominates. Similarly, the authors of Schoenfeld [number of connections: 87, total connection strength: 1300] and Freudenthal [number of connections: 75, total connection strength: 637] also have strong connections. In the purple cluster, authors Niss [number of connections: 89, total connection strength: 3054], Blomhoj [number of connections: 86, total connection strength: 2198], and Frejd [number of connections: 82, total connection strength: 1133] have strong relationships. Finally, in the light blue cluster, Sriraman [number of connections: 81, total connection strength: 1044] came to the fore regarding connection strength. Below is a network visualization of co-citation analysis in the context of cited journals (Figure 8).

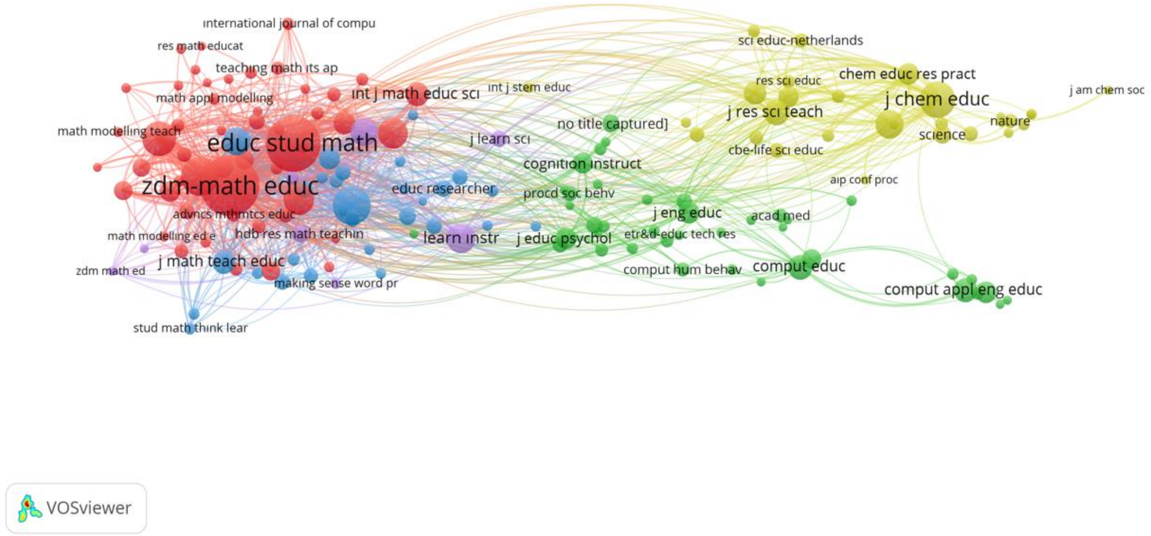


Figure 8. Co-cited network analysis in the context of cited journals

In Figure 8, the articles published on the subject of MM have been preferred so that the minimum number of citations of one cited journal is twenty. According to this threshold value, the total number of journals cited from the studies in the data set made the analysis 11712, and the number of cited studies meeting the threshold value is 156. It is seen in the figure that five different colored clusters are formed. The journals ZDM Mathematics Education and Educational Studies in Mathematics come to the fore in the red cluster. Journal of Educational Psychology and Cognition Instruction are dominant in the green cluster. The Journal for Research in Mathematics Education and Journal of Mathematica Teacher Education have stronger links in the blue cluster. The Journal of Chemical Education and Research Science Teaching is more prominent in the yellow cluster. Learning and Instruction and Journal of Mathematical Behavior are more dominant in the purple cluster. These journals are core journals and are central to more networks. The figure below shows the co-author network analysis in the context of the authors (Figure 9).

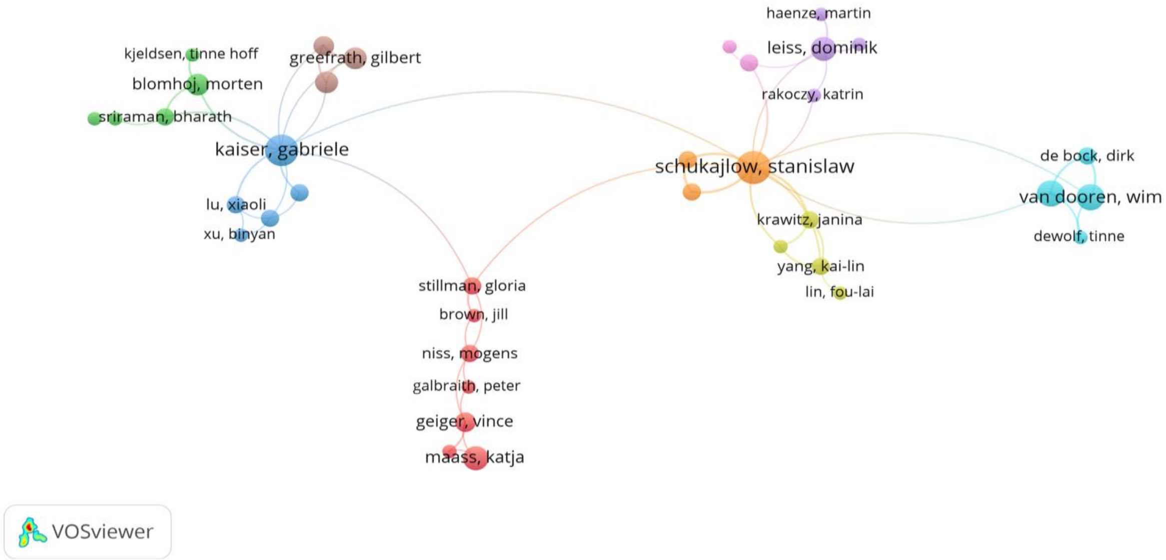


Figure 9. Co-author network analysis of articles in the context of authors

In Figure 9, choose the minimum number of studies by an author as two to determine the co-author network analysis of published articles on MM in the context of authors. Kaiser dominates the blue-colored cluster. Lu, Xu, Schwarz, and Yang are other authors in this cluster. Schukajlow dominates the orange-colored cluster. Leiss dominated the purple cluster, Blomhoj dominated the green-colored cluster, and Van Dooren and Verschaffel dominated the light blue-colored cluster. According to the figure, it is seen that the cooperation between the authors is limited. When both the connection and the node size are examined, it is seen that there is a limited number of collaborations between the authors. Below is the density map for co-author collaboration in the context of institutions (Figure 10).

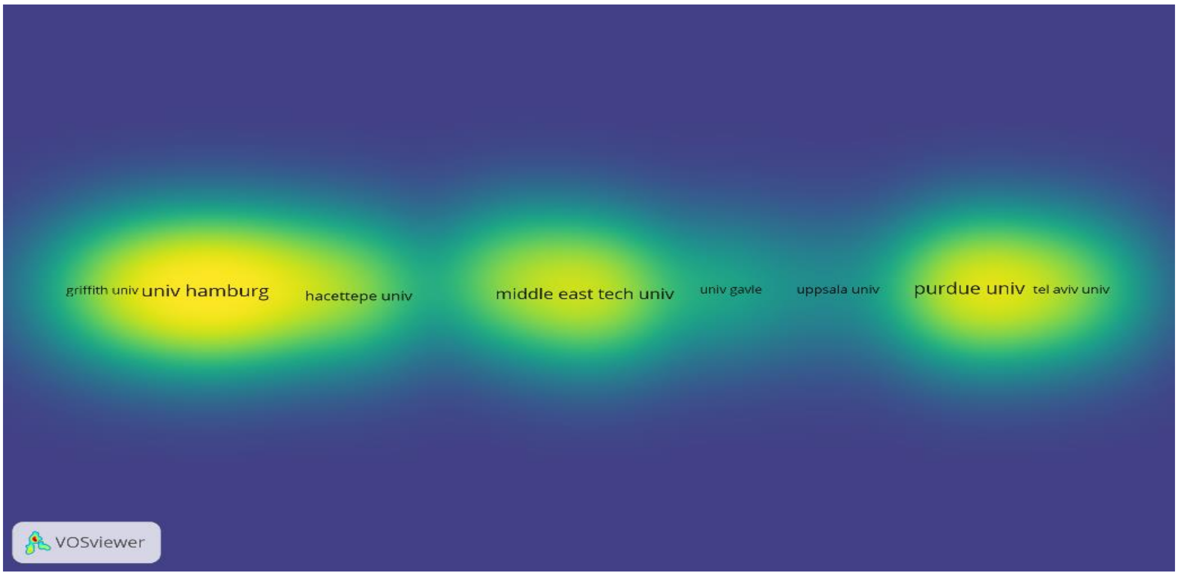


Figure 10. Co-author density map of articles in the context of institutions

Figure 10 shows the co-author density map representation in the context of institutions. According to the density map created for cooperation at the level of institutions, it is seen that the collaboration between institutions remains at a limited group. Although there is little entity between Hacettepe University and Middle East Technical University, Hamburg and Purdue Universities have limited cooperation. Therefore, the sparse collaborative work on MM indicates the need for more collaboration between countries. In the figure below, the result of the network analysis for co-author cooperation in the context of countries on MM is given (Figure 11).

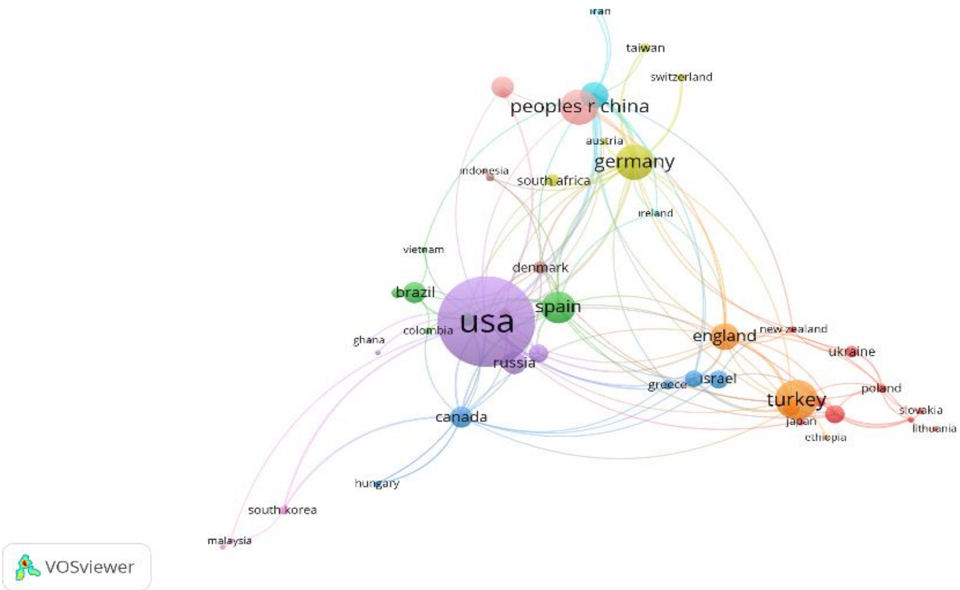


Figure 11. Co-author network analysis of articles in context of countries

Figure 11 shows the network analysis of published research on MM among 60 countries collaborating in scientific production. The figure shows that USA, Turkey, Germany, and China stand out as the most productive countries. The purple cluster with the USA also includes the countries of Ghana, Mexico, Scotland, and Russia. The orange-colored cluster containing Turkey includes England and Norway. The yellow cluster containing Germany includes Switzerland, Taiwan, South Africa, and Australia. A closer look at the figure shows that some collaborative initiatives between countries stem from scientific productivity rather than geographic and linguistic proximity. The world cooperation network is shown in the figure below (Figure 12).

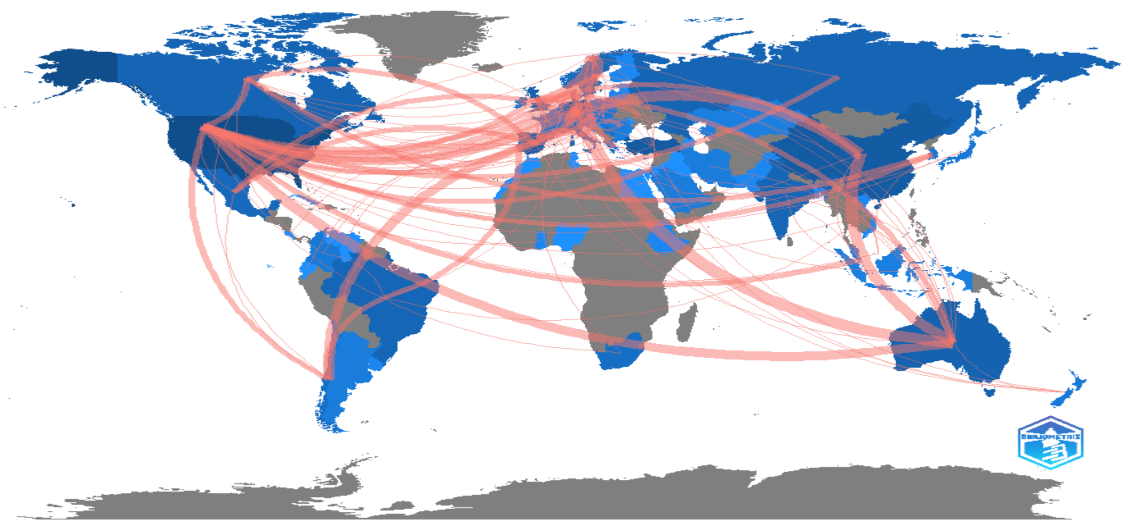


Figure 12. The MM authors' world collaboration network

Figure 12 shows the cooperation between countries more clearly. Relationships between China and Germany, Germany and Australia, China and Australia, England and Norway, Spain and Chile, Sweden and Norway, and USA and Germany are noteworthy. Similarly, there are partnerships between Germany and Switzerland, England and Sweden.

Keywords and Co-Occurences Network Analysis

Because of their abstract nature, they can use keywords to reveal the content of an article. The following figure shows the word cloud of the studies published on MM based on keywords (Figure 13). A word cloud is a visual tool for summarizing textual data. The size of each term and its proximity to the cloud center determine its importance (Liao, Tang, Li & Lev, 2019).

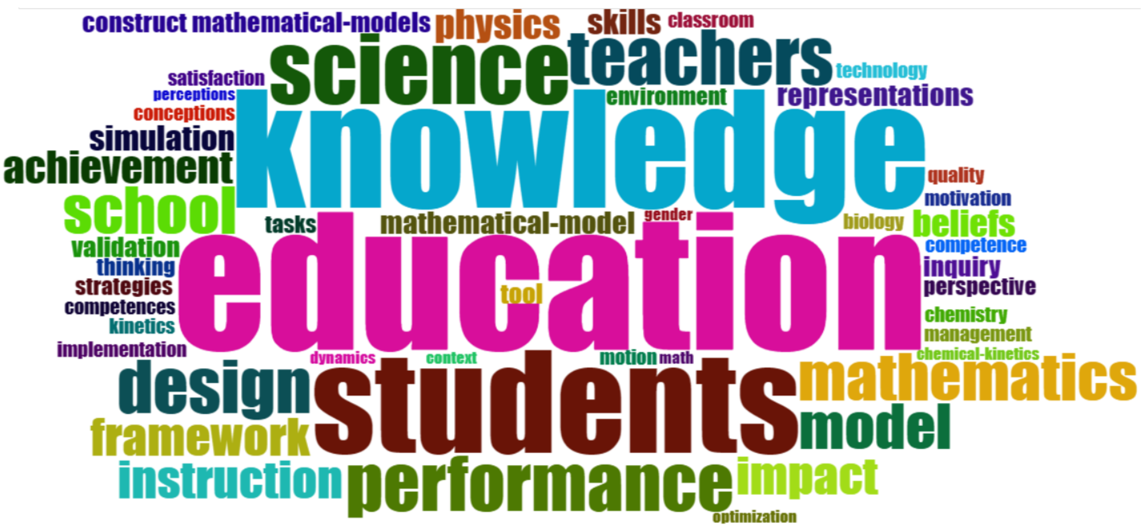


Figure 13. Word cloud for MM

When the word cloud in Figure 13 is examined, the most frequently used keywords are education, knowledge, students, science, performance, teachers, design, mathematics, school, and model appears. Common word network analysis examined how often keywords are used in the same document. In this context, examined the co-occurrence of the keywords created by the authors. In this way, the relationships between the keywords determined in an article were tried to be determined. The following figure shows the common word network analysis, in other words, the occurrences between keywords (Figure 14).

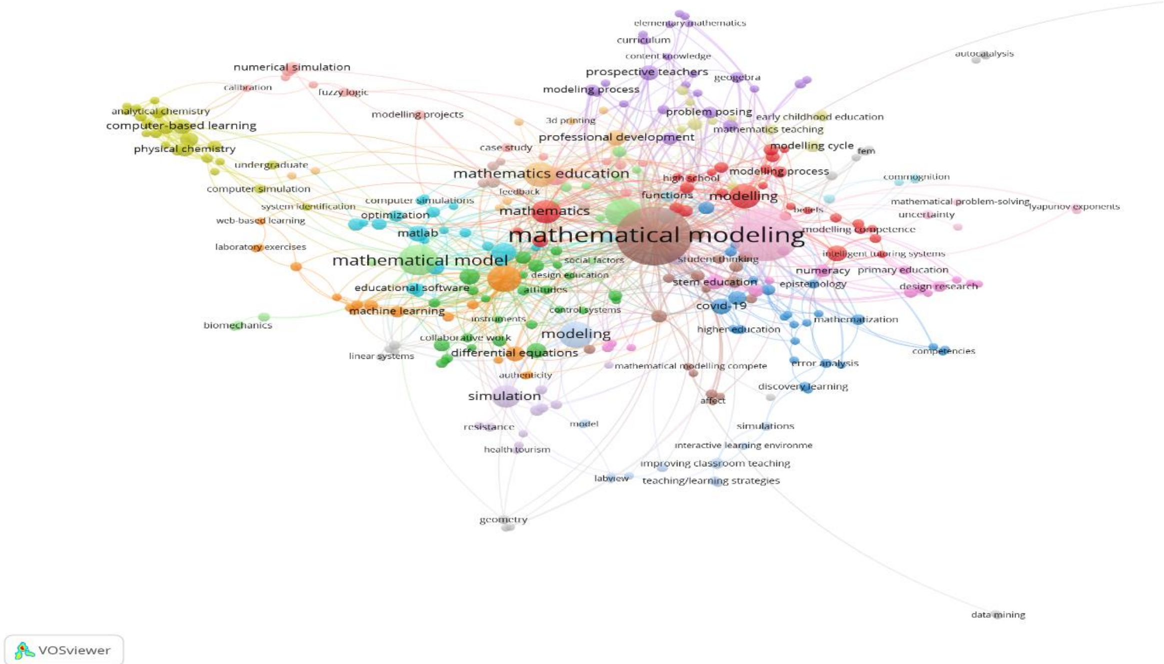


Figure 14. The MM author-provided keyword co-occurences network

When Figure 14 is examined, the brown cluster includes words such as mathematical modelling, self-efficacy, metacognition, and pre-service teachers. The pink cluster includes words such as mathematical modelling, mathematical problem-solving, and numerical investigation. The green-colored cluster includes words such as mathematical models, reasoning, problem-solving, and cognitive development. The purple-colored cluster includes words such as prospective teachers, problem posing, modelling process, and curriculum. In the figure below, the three-domain diagram, also known as the Sankey diagram, links to each other by keywords (left), authors (middle), and institutions (right) to contextualize the MM flow trend (Figure 15).

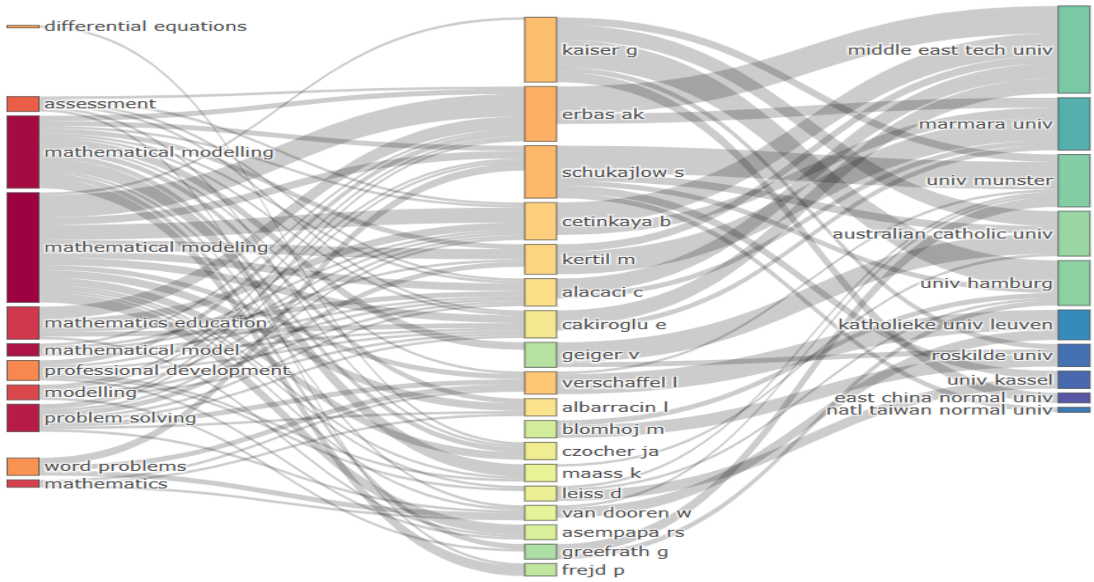


Figure 15. The MM Sankey Plot (keyword- author-affiliation)

When Figure 15 is examined, the box size is proportional to the number of links (keyword, author, or institution). According to the figure, keywords such as "mathematical modelling", "mathematical modelling", "mathematics education" have the most significant margin widths. The large margins show that many authors use keywords in their publications. On the other hand, it is noteworthy that Kaiser, Erbas, and Schukajlow authors are used in a vast list. Also, some authors use a comprehensive keyword list reflecting the diversity of their research (Kaiser), while others (Frejd) use a unique set of keywords.

Trending Topics and Thematic Evolution

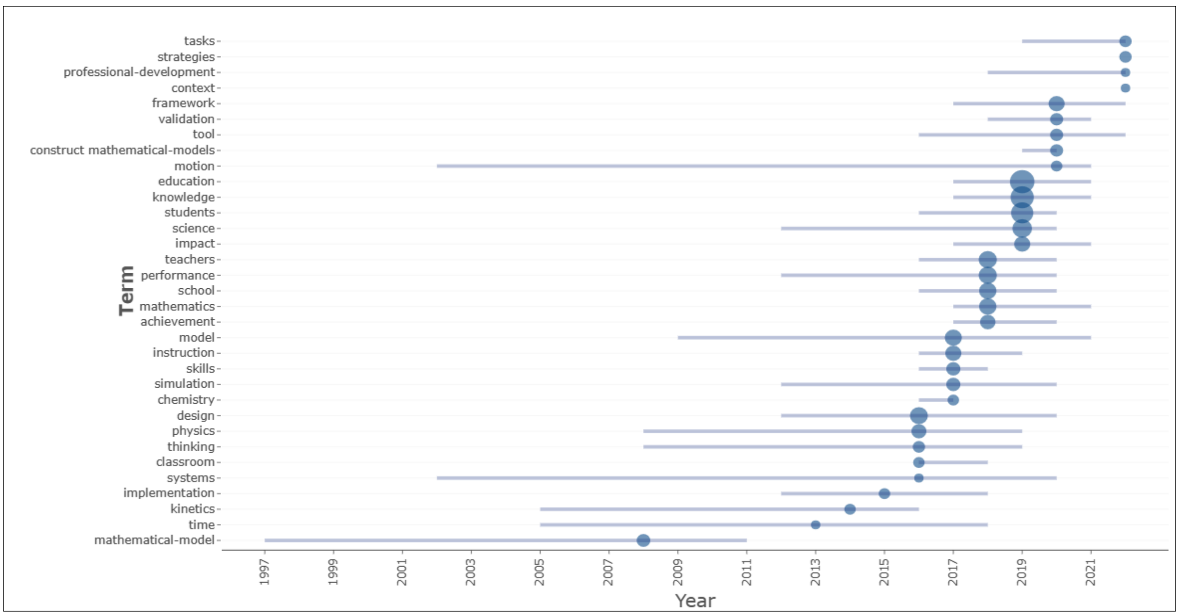


Figure 16. The MM trending topics map (keywords plus)

Figure 16 shows the main MM train topics. According to the figure, it is seen that there is a transition from established issues such as the "mathematical model" (1997-2011), and "time" (2005-2018) to "tasks", "strategies", and "professional development" (after 2018). Such topics are considered "trending topics/hotspots" in scientific publications related to MM. Because trending topics are often used to represent emerging themes in a particular field of research, they can also be seen as a series of increasing importance in the knowledge base in a specific discipline.

Conceptual Structure and Thematic Maps

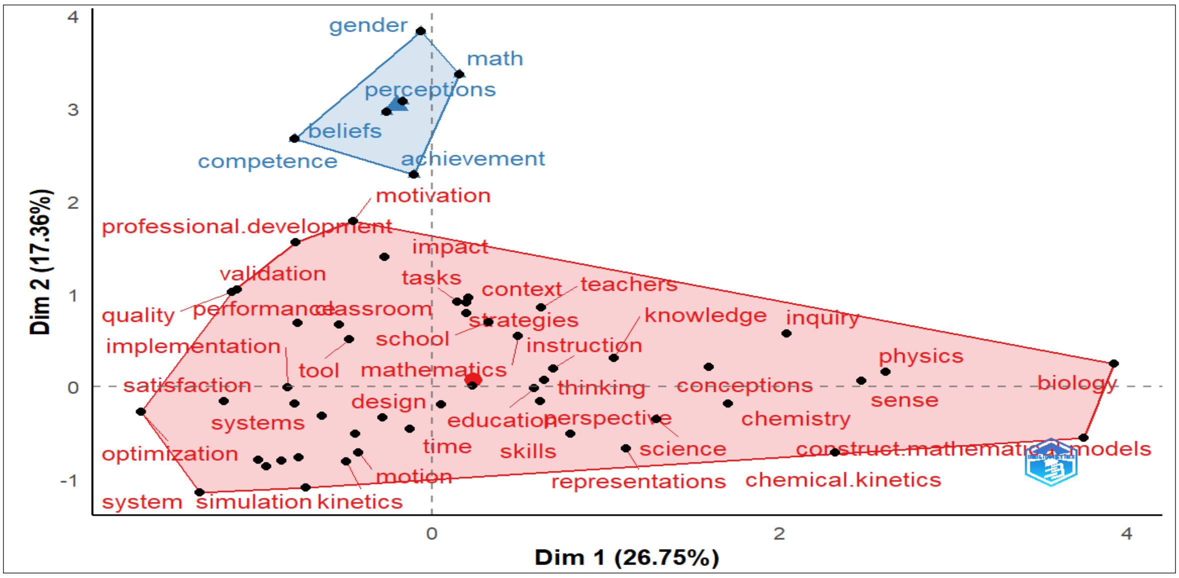


Figure 17. The MM conceptual structure map (MCA method)

Figure 17 shows the conceptual map generated by the MCA method on keywords. At the same time, this map reveals the conceptual structure of 40 years of MM. According to the graph, the best size reduction obtained for the first two dimensions of the MCA accounts for approximately 44% of the total variability. In this graph, the closer the points are to each other, the more similar the profile they represent, and each cluster of points represents a distinctive profile (Wong, Mittas, Arvanitou & Li, 2021). Examining the graph reveals the depth and breadth of the MM conceptual structure. For example, the most prominent red cluster contains keywords that highlight different areas of interactive work, such as "thinking", "instruction", "knowledge", "performance", "strategies", "teachers", and "context". The second cluster (blue) is more of a psychological learning dimension related to MM. This cluster includes keywords such as "perceptions", and "beliefs". The thematic/strategic map is given in the figure below (Figure 18).

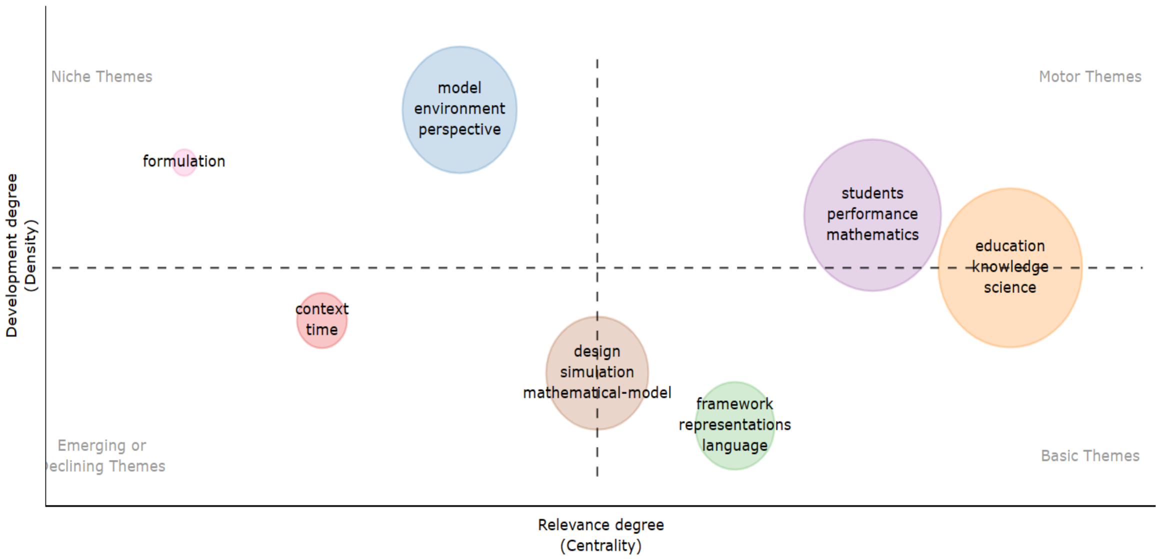


Figure 18. The MM thematic map

Figure 18 shows the thematic/strategic map of MM. In the graph, the average values of both axes are represented by a dotted line dividing the map into four quadrants. While each quadrant in this chart represents a different theme, the bubble size is drawn in proportion to the frequency of the documents in which the keywords are used. The first quarter of the graph represents well-developed motor themes internally and externally, as it is characterized by high intensity and centrality (Cobo, Lopez-Herrera, Herrera-Viedma & Herrera, 2011). Engine themes involving MM include "students", "performance", "mathematics", and "education". On the other hand, the second quarter is known as Niche themes, which are very developed and isolated. This theme exhibits a high density and low centrality structure. The content of this much is based on the fact that while it is well developed internally, it is of little importance externally. Niche themes involving MM include "model", "environment", and "perspective". The themes in the third quarter include low density and low centrality. Weak ties at the inner and outer levels characterized the themes in this quadrant. Such themes indicate potential hotspots in research on interactive learning environments. Examples include "context", "time", "design", "simulation", and "mathematical model". Finally, the "core and cross themes" quarter (low density-high centrality) included poorly developed themes in terms of interconnections. However, they are characterized by significant external bonds. Genre themes involving MM have "framework", "representations", "mathematical model", "design", and "simulation".

Discussion and Conclusion

This study has tried to trace the knowledge structure of the articles published on MM and to determine the comprehensive view of the scientific articles published on MM between 1981 and 2022. In this context, first examined the distribution of scientific articles published on MM in the WoS database by years and the number of citations. According to the findings, it has been determined that the articles published on MM have changed over the years. It is seen that the articles published on MM from 1981 to the end of 2022 are grouped under three categories. The first category is between 1981 and 2005. There has not been a significant increase in the number of articles published on MM between these years. The second category is between 2006 and 2016. Although there was no significant increase in the number of articles published between these years, it is noteworthy that between 28 and 40 articles were published. The third category is between 2017 and 2022. Notably, there has been a significant increase in MM in this category. In particular, the studies published on MM in 2021 reached their peak. It is noteworthy that there is a decrease in this category in 2020. This situation is thought to be caused by the effect of the pandemic. On the other hand, when the number of citations on MM is examined, it is seen that the number of citations tends to increase continuously depending on the increase in the number of articles. When these findings are evaluated in terms of scientific production, it is seen that the subject of MM has become more popular as it approaches today. It can say that studies on MM will tend to increase in the coming years, depending on the richness of the content of scientific approaches, especially in conceptual contexts. It is frequently emphasized that MM should become an important skill for future generations, and it is stated that it is necessary to focus on studies in this direction (Blum, 2015; NCTM, 2014).

When the published studies on MM based on the WoS database are examined, it is seen that the most productive authors are Erbas, A. K., Schukajlow, S., Kaiser, G., and Verschaffel, L. When the general study profiles of these authors are examined, it is seen that they have publications in quality journals on MM. According to WoS author information, the institutions where these authors work are Middle East Technical University (Turkey), University of Münster (Germany), Australian Catholic University (Australia), and Leuven University (Belgium). The work of these authors, who are the pioneers of the core authors, both directs the studies in the field and contributes to a better understanding of the theoretical foundations of the subject of MM. The study "applying covariational reasoning while modelling dynamic events: a framework and a study", published by Carlson et al. in 2002, has been the most cited study in the related field. The study aims to propose a framework to explain the mental actions involved in covariational reasoning. The article "modelling reality in mathematics classrooms: the case of word problems", written by Greer, B. in 1997, is among the most cited articles. In this study, mathematical modelling processes based on real-life knowledge are discussed in the context of verbal problems. Another most cited study was the article titled "predicting student academic performance in an engineering dynamics course: a comparison of four types of predictive mathematical models", written by Huang, S. and Fang, N. in 2013. In this study, the authors consider students' academic performance based on four mathematical models. Among the standard features of the most cited studies is that they contain content on how modelling can be done at a time when the concept of mathematical modelling is just beginning to take shape. In this respect, these studies have gained an important place in the field and have guided researchers in modelling. Although the studies on modelling are not sufficient, it is seen that there has been a significant increase in the number of studies on this subject in recent years (Cevikbas et al., 2021).

When the authors' countries working on MM were examined, it was determined that the USA came to the fore. Turkey, Germany, China, Spain, and England follow this country. The remarkable detail in this part of the research is that the studies were written mainly by authors from a single country. In this case, it shows that multi-country author cooperation is minimal. While Germany came to the fore in collaborating with multi-country authors, Russia remained in the background. When the findings about the authors who have been dominant in MM from the past to the present are examined, it has been determined that some authors have come to the fore at certain time intervals. For example, Erbas from 2010 to 2021, Kaiser from 2006 to 2022, Schukajlow from 2010 to 2022, and Verschaffel from 1997 to 2020 played a role as dominant authors. In summary, these authors have gained an important position in the field of MM in the last twenty years and have made significant contributions to the area with their published studies. On the other hand, some short-term dominant authors have left their mark on MM from the past to the present. For example, Blomhoj from 2006 to 2013, Çakıroğlu from 2017 to 2021, and Alacacı from 2014 to 2017. The most productive institutions in MM are Purdue University, followed by Australian Catholic University, Münster University, Middle East Technical University and Kazan Fed University, respectively. The most important reason for these institutions to come to the fore is that most authors who publish on MM work in these institutions. So this is an expected result. Looking at the geographical atlas of countries' scientific production, the USA, China, and Turkey produced more documents on MM. Although these countries are the leading countries in the field of MM, many scientific productions have been carried out in countries such as Spain, Germany, and Russia. The remarkable point of this finding is that scientific production has taken place on the subject of MM from many continents. Many researchers in the related literature offer different perspectives on the field with valuable studies and propose approaches that form the starting point of many studies on MM (Blum, 2011; English et al., 2005; Erbas et al., 2014; Kaiser, 2020; Maaβ, 2006; Sokolowski, 2015; Verschaffel et al., 2002). Therefore, the interest of researchers from different countries in MM increases the prevalence of studies in this field.

Another study finding was obtained from the co-citation analyses of the articles related to the subject of MM. A co-citation network occurred when two authors cited a third reference. The analysis criteria determined that specific authors are centrally located in six clusters. In particular, it determined that the authors of Lesh, Doerr, English, Gainsburg, Cobb, Ärlebäck, Lehrer, Gravemeijer, Blum, Kaiser, Borromeo-Ferri, Stillman, Maass, Verschaffel, Schoenfeld, Freudenthal, Niss, Blomhoj, and Sriraman are in the center of their cluster. In particular, the knots between Lesh, Doerr, and English authors were found to be thick and frequent. Similarly, the knots between Blum, Kaiser, Mass, and Stillman are thick and tight. Therefore, these authors united and discussed the axis of a common subject. These similarities that emerge in bibliometric analyses indicate intradisciplinary or thematic similarity (Jiang, Ritchie & Benckendorff, 2019). Centrally located in each cluster authors are more likely to influence other communities. Also, these authors significantly affect other communities as they control and encourage the dissemination of knowledge through their research activities (Mostafa, 2020). Therefore, they are considered influential authors related to the field of study. Another finding of the study was obtained from commonly cited sources. When analyzed according to the determined criteria, five different clusters were obtained. In these clusters, ZDM Mathematics Education, Educational Studies in Mathematics, Journal of Educational Psychology, Cognition Instruction, Journal for Research in Mathematics Education, Journal of Mathematica Teacher Education, Journal of Chemical Education, Journal of Research Science Teaching, Learning and Instruction and Journal of Mathematical Behavior journals are more dominant. These journals came to the forefront as "core journals" in their clusters. These journals tend to be heavily cited on MM. In other words, it has an "orthodox core-journal" feature (Dobusch & Kapeller, 2012; Glotzl & Aigner, 2018). Journals that are pioneers in the field of MM have an important position both in getting more citations and directing the relevant field. On the other hand, one of the remarkable features of these journals is that the authors who stand out on the subject of MM continue their publishing lives in their countries. For example, the journal ZDM Mathematics Education is published in Germany, and most of the prominent authors on MM work in institutions in Germany.

Another study finding was obtained from co-author network analysis in the context of the authors. When analyzed according to the determined criteria, Kaiser, Schukajlow, Leiss, Blomhoj, Van Dooren, and Verschaffel are the authors who dominated their clusters. It was determined that Kaiser, Schukajlow, Blomhoj, Geiger, Van Dooren, and Mass were the authors who assumed the role of information dissector by acting as a bridge between other authors. Therefore, these authors are called information brokers (Park, Lim & Park, 2015). According to the findings, there are also authors with a sparse network and a limited number of collaborations. For example, Dewolf, Greefrath, and Kjeldsen are some authors. These authors are isolated but influential researchers on MM (Mostafa, 2022; Vidgen, Henneberg & Naude, 2007). Another study finding was obtained from the co-author density map in the context of institutions. Accordingly, it has been determined that Hamburg, Hacettepe, Middle East Technical, and Purdue Universities have more collaborative initiatives at terms of institutional level. Although a limited number of collaborations have been made, these institutions stand out from the center cooperation network. When the cooperation network is examined in the context of countries, it is determined that there is cooperation between 60 countries. Especially USA, Germany, and Turkey have come to the fore as highly productive countries in MM. According to the geographical atlas, it has been determined that productivity is a more critical factor in cooperation between countries rather than geographical and linguistic proximity.

Another finding was obtained from the word cloud of the studies published on MM based on keywords. Conclusions of the word cloud showed that preferred the keywords education, knowledge, students, science, performance, teachers, design, mathematics, school, and model. Due to their abstract nature, keywords provide essential clues in determining the content of articles (Chen, Zou, Xie & Cheng, 2021). The co-occurrence network of keywords was also examined to determine how often keywords co-occur in the same document. Accordingly, mathematical modeling, self-efficacy, metacognition, pre-service teachers, mathematical modeling, mathematical problem-solving, numerical investigation, mathematical model, reasoning, problem-solving, cognitive development, prospective teachers, problem posing, modeling process, and curriculum words are at the center of the clusters in which they are found. In the findings obtained from the three-domain diagram known as the Sankey diagram, keywords such as mathematical modelling, mathematical modelling, and mathematics education are foregrounded. This indicates that many authors frequently use these keywords. In addition, it determined that some authors used a comprehensive keyword list reflecting the diversity of their research (Kaiser, Erbas). In contrast, some authors used unique keywords (Frejd, Greefrath).

When the study findings related to the trending contents of MM are examined, it is seen that the established contents, such as "mathematical model" (1997-2011) and "time" (2005-2018), have changed to contents, such as "tasks", "strategies", and "professional-development" (after 2018). Such content is considered as "trending topics/hotspots" in scientific publications. Because trending topics often represent emerging themes in a particular research area (Chen et al., 2021; Mostafa, 2022). It can also be seen as a series of increasing importance in knowledge in a particular discipline (Qian, Law & Wei, 2019). In this context, there has been a significant increase in tasks and strategy-related content. Scientific studies based on MM tend to be professional development. The findings of many studies today center on the competence framework of students in MM (Blum, 2011, 2015; Kaiser, 2015; Lesh & Doerr, 2003; Xu et al., 2022). Developing technology can be cited as one of the most important reasons why trending content has periodic effects. Therefore, depending on the innovations brought by today's conditions, MM differs in the changes in the content of the subject. Trending topics also refer to know-how within a particular discipline. In this respect, the trending content of the issue of MM today provides essential clues in determining the themes of similar studies to be made. Another research finding was obtained from the conceptual map on which the MCA method was applied to the keywords. The determined conceptual map reveals the forty-year-old MM conceptual structure. The results determined that the best size reduction obtained for the first two dimensions of MCA constituted approximately 44% of the total variability. Accordingly, it determined that the most significant cluster that emerged included keywords emphasizing interactive work areas such as thinking, performance, instruction, knowledge, strategies, teachers, and context. Another cluster reveals more psychological learning dimensions related to MM. This cluster has been determined to contain keywords such as perceptions and beliefs. While the close representation of the keywords that make up the cluster shows that the cluster has similar properties, the representation power decreases as the distance between the points increases (Wong et al., 2021).

The last study finding of the research was obtained from the thematic/strategic mapping of MM. Accordingly, the motor themes include students, performance, mathematics, and education. Since this structure is characterized by high density and centrality, it represents well-developed internal and external themes (Cobo et al., 2011). In other words, the themes of performance, mathematics, and education constitute important study content of MM. These themes have taken an important place in many studies' starting points. On the other hand, model, environment, and perspective are essential as Niche themes. These themes are very advanced and isolated. Although these themes are frequently included in the content of the MM subject, they also constitute a large community. Low-density and low-centered themes include context, time, design, simulation, and mathematical models. The most distinctive feature of these themes is that they represent hot spots in the research of learning environments and are the themes that researchers focus on (Mostafa, 2022). In addition to these, there are also themes characterized by external links. Core and cross themes include poorly developed themes in terms of internal ties. However, these themes are marked by external and solid relations. For example, framework, representations, mathematical model, design, simulation, and language themes form groups of themes characterized by external links on MM.

Recommendations

Within the scope of the study, the subject of MM was taken as the basis. In studies to be conducted in a similar direction, different subject contents can be included. WoS database was used in the selection of scientific articles published on MM. Scopus, Eric, *etc.*, databases can reach more datasets in similar studies. In addition, changes can be made in the search criteria in similar studies to be carried out. For example, SSCI, SSCI Expanded, ESCI, and A&HCI indexes were preferred in the survey. However, Conference Proceedings Citations Index-Social Science & Humanities (CPCI-SSH) and Conference Proceedings Citations Index-Science (CPCI-S) categories can also be preferred in WoS Index information. In this way, not only articles but also other documents can be included in the studies. In addition, English articles as a general language choice also formed the study's data set. Different languages can be included in the data set in published scientific studies. Therefore, by creating broader search criteria, the findings of this study can be tested with a larger-scale data set. Future studies may continue to analyze published research articles on MM after 2023. This way, MM changes can be better monitored over the years. Finally, a more comprehensive analysis can be performed with the help of content analysis at the point of the dynamic changes in MM over time. This study discussed scientific productivity, network analysis, conceptual structures, thematic maps, and trends. Therefore, a more in-depth roadmap can be created with the help of content or descriptive research.

Limitations and Future Research

The study has some limitations as well as several contributions to the field. One of the most critical limitations of the study is that only the WoS database was preferred for accessing scientific articles published on MM. Another limitation of the study is that the scientific articles included were written in English. Therefore, the contents of articles written in other languages (Germany, Spanish, French, Turkey, *etc.*) may differ. In addition, articles were selected as the document type in the study. Document types such as early access, review article, proceeding paper, book review, and editorial material can be used in similar studies. Another limitation of the study is that only Education & Educational Research and Education Scientific Disciplines categories were selected as WoS category preference. There may be differences in the content of scientific articles outside these categories.

Acknowledgement

Due to the scope and method of the study, ethics committee permission was not required.

Author Contribution Statement

Tamer KUTLUCA: *Conceptualization, literature review, methodology, implementation, data analysis, translation, writing, auditing, and editing processes.*

Deniz KAYA: *Conceptualization, literature review, methodology, implementation, data analysis, translation, writing, auditing, and editing processes.*

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